Lecture 6

Monday, January 25, 2021 2:11 PM

* Prayer

* Spiritual thought: "lift where you stand" – Elder Uchtdorf.

Each person is needed, and if we do our part, the work will move along.

* Answering questions ....

Vector functions: $f(t) = \langle g(t), h(t), \ldots \rangle$

(functions with vector values)

A curve is a vector function.

Recall that a curve is more than just a shape. It must tell us how it is drawn (i.e. parametrized).

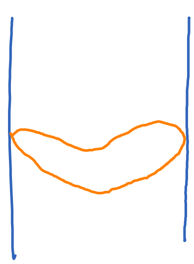
Ex: Find the intersection of two surfaces

$x^2+y^2=4$, $z = xy$

$x = 2\cos t$

$y = 2\sin t$

$z = 4\cos t \sin t$ $(= 2\sin 2t)$



Ex Find the intersection of two surfaces $\underbrace{z=\sqrt{x^2+y^2}}_{\text{cone}}$, $\underbrace{z=1+y}_{\text{plane}}$

$y=t$

$z=1+t$

$x=\sqrt{z^2-y^2}=\sqrt{(1+t)^2-t^2}=\sqrt{2t+1}$ $\quad\left(t\geq -\frac{1}{2}\right)$

Ex

$$\begin{cases} x^2-y^2=1 & \text{(hyperbolic cylinder)} \\ x^2+z^2=4 & \text{(circular cylinder)} \end{cases} \rightsquigarrow y^2+z^2=3$$

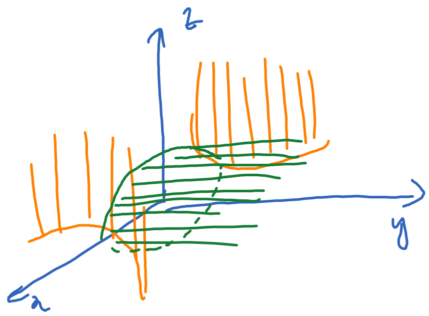
$y=\sqrt{3}\cos t$, $z=\sqrt{3}\sin t$

$x=\sqrt{1+3\cos^2 t}$

~~$x=2\cos t$~~

~~$z=2\sin t$~~

~~$y^2=x^2-1=4\cos^2 t-1 \rightsquigarrow y=\pm\sqrt{4\cos^2 t-1}$~~



$y=t$

$x^2=1+t^2 \rightsquigarrow x=\pm\sqrt{1+t^2}$

$z^2=4-x^2=3-t^2$

$$\begin{cases} x^2=1+y^2 \\ z^2=4-x^2=3-y^2 \end{cases}$$

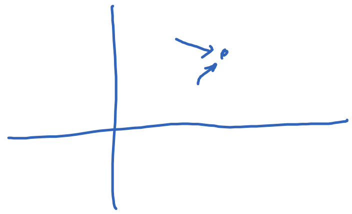
Ex: $x^2+y^2+4z^2=4$

$x^2+z^2=1$

Limit

$$f(t) = \langle g(t), h(t), \ldots \rangle$$

$$\lim_{t \to t_0} f(t) = \langle \lim_{t \to t_0} g(t), \lim_{t \to t_0} h(t), \ldots \rangle$$



Derivatives

$$f'(t) = \langle f_1'(t), f_2'(t), \ldots \rangle$$

why?

$$f'(t) = \lim_{h \to 0} \frac{f(t+h) - f(t)}{h} = \lim_{h \to 0} \left\langle \frac{f_1(t+h) - f_1(t)}{h}, \ldots \right\rangle$$

$$= \langle f_1'(t), \ldots \rangle$$

Integral...

Ex Find unit tangent vector of the curve

$$r(t) = \langle \sin t, 5t, \cos t \rangle \quad \text{at} \quad (0, 0, 1)$$

Find $\displaystyle\int_{-\pi/2}^{\pi/2} r(t)\, dt$,